Aalto University
School of Science

In collaboration with
Danske Markets

Mat-2.4177 – Seminar on Case Studies in Operations Research

Modeling Long-Term Electricity Prices

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Midterm Report

Abstract

This midterm report presents the current status of our project and changes in our approach as well as our results this far. Some discussion about VAR and CVAR models is also included. The status of the project is also reviewed with respect to possible risks and what has been done this far.

Overall Project Status

As explained in our project plan, our final goal is to model and forecast the long-term prices of electricity forward contracts. We try to model the dynamics of electricity prices with VAR (vector autoregressive) and CVAR (cointegrated vector autoregressive) models. This approach is based on the paper Povh and Fleten (2009)

The original approach focused mainly on the mathematical formulation of the model and a literature review. After some useful questions from our client, we have decided to add some more practical and general discussion to our study. The prices for the annual forward contracts are not changing linearly so we are also interested in their relative prices across the forward curve.

In addition, changes in market conditions or technological and economic environment affect the model significantly. These phenomena will be discussed with respect to both our model and general features of the electricity market. One of the nearest and most important is the European Union Emission Trading Scheme which was launched in 2005 and will expand considerably largely in 2013 to prevent CO_2. After discussing with the course personnel, we have also paid attention to the long-term perspective of the production and consumption of the electricity in the time horizon of decades rather than only a couple of years.

Because we found the articles of Povh and Fleten very useful and important to our study, we contacted them via email. Especially the doctoral thesis of Povh (2009) treated more deeply the same topics as the Povh and Fleten (2009) article.

Materials and Methods

At this stage of the project we have all the basic material and tools for doing the rest of the project. Most importantly, we now have actual data and working codes for model estimation.

(a) Model Structure

The vector autoregressive model (VAR) is a extension of the basic AR-models where all variables are treated endogenously. The basic formulation of kth order VAR model with external variables is

$$y_t = A_0 + \sum_{i=1}^{k} A_i y_{t-i} + \sum_{j=1}^{m} B_j x_{t-j} + \varepsilon_t, \tag{1}$$

where $y_t \in \mathbb{R}^n$ is a vector of endogenous variables at time step t and $x_t \in \mathbb{R}^s$ is a vector of exogenous variables. $A_0 \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times s}$ are coefficient matrices and $\varepsilon_t \in \mathbb{R}^n$ is the error term. The estimation of this model can be done in Matlab with the *Econometrics Toolbox* that is included in the default setup.

The cointegrated vector autoregressive (CVAR) model or a vector error correction

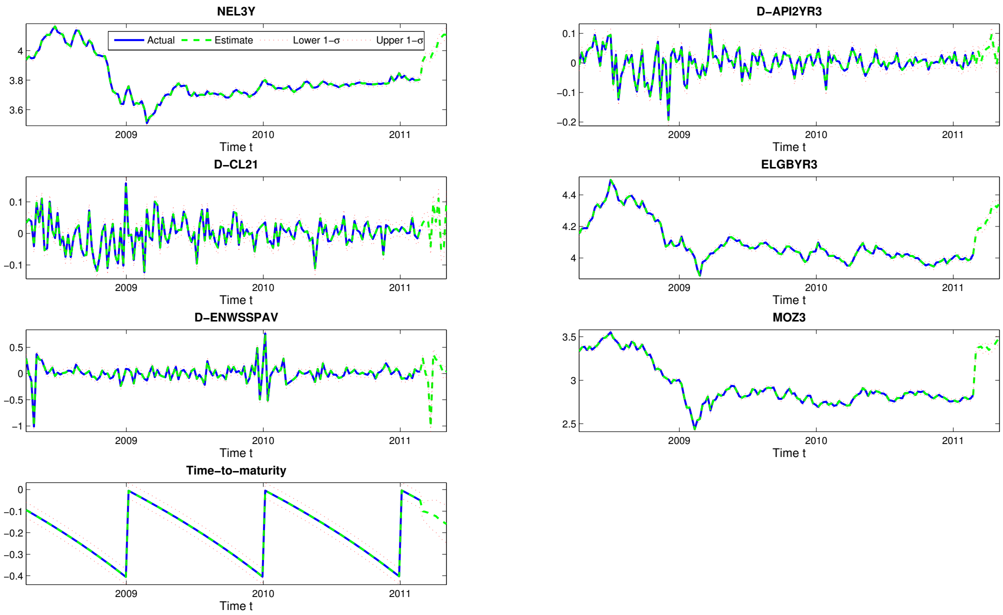

Figure 1: A VAR model of order 2 with seven endogenous variables. The prefix D refers to differentiation of the data to obtain stationarity.

model (VECM) is a extension to handle cointegrated variables. The model can be formulated as

$$\Delta y_t = A_0 + \Pi y_{t-1} + \sum_{i=1}^{k-1} A_i \Delta y_{t-i} + \sum_{j=1}^{m} B_j \Delta x_{t-j} + \epsilon_t, \qquad (2)$$

where $\Delta y_t = y_t - y_{t-1}$ corresponds to the differentiated variables and $\Pi \in \mathbb{R}^{n \times n}$ is the error correction matrix. Estimating a model like this is not as straight-forward as estimating a basic VAR model. For this reason we have used a ready-made *Econometrics Toolbox* by *James P. LeSage*. All relevant tests and estimation can be done with the help of these two toolboxes in Matlab.

(b) Datasets

We have acquired some data from Danske Markets. The data includes the Nordic and German electricity forward prices spanning some 5 years. Additionally we have also received electricity spot prices, crude oil and coal forward prices and emission allowance prices. The data has been downsampled to week level so that the week mean prices is taken from the Wednesday closing price — or the nearest price available.

The same prices are present in the data twice. In the first set of prices we have the actual contract price time series. In the second set the different contracts of the same product have been combined into generic datasets. In these datasets the time to maturity is fixed

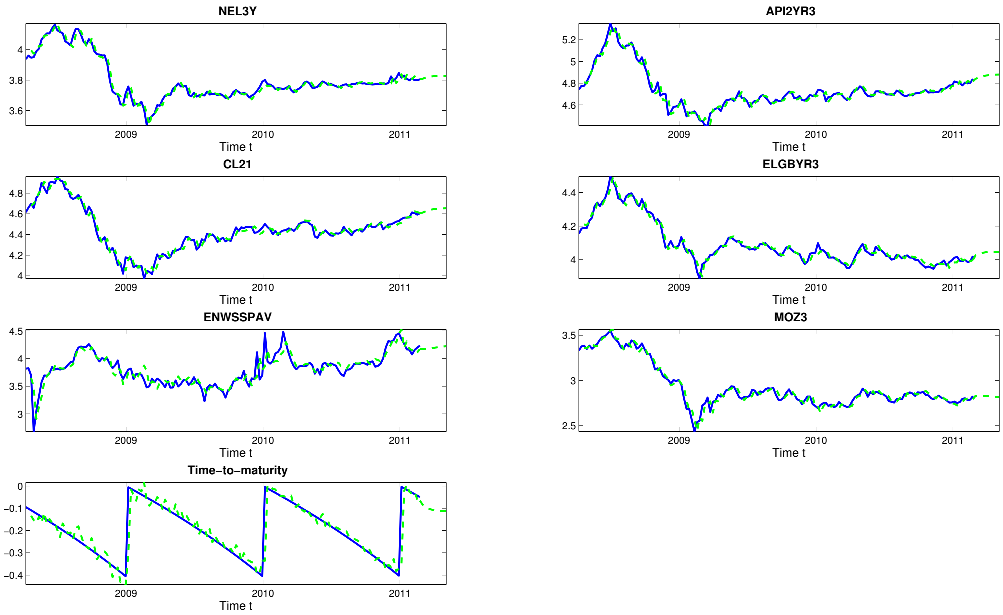

Figure 2: A 1st order CVAR model estimate with seven endogenous variables. The dashed green line is the estimate with a 10 step forecast and the solid blue line the original data.

within each set. For instance the third year generic data for the electricity forwards (NEL3Y) consists of the consecutive contracts and data for each year is always the third nearest annual contracts for that specific year.

(c) Results and Findings

At this stage of the project we have some preliminary results from both the VAR and CVAR models. To achieve as long datasets as possible we decided to use generic data when possible.

In addition of generic electricity forward prices we are we are using forward prices of coal (API2YR3) and oil (CL21), the German electricity forward price (ELGBYR3), spot price (ENWSSPAV) and European union emission allowance prices (MOZ3) as endogenous variables in our model. The focus has been on formulating and achieving general understanding of the model so the particular choice of variables has not been considered yet.

The Figure 1 presents the estimate and the forecast of the third year contracts with a VAR model. The fundamental variable we are interested in is the forward price of the electricity (NEL3Y) which is depicted in the upper most graph. Because we are using the generic data, the datasets are not fully continuous. Thus we decided to add an external variable (time to maturity) to depict the change of the contract to the model. This saw-like variable gets a value one when a contract changes in the dataset and the value of the external variable goes linearly to zero until a new contract emerges to the

Risks		Current status of risk	Probability	Severity
A	**Quality of data**			
A1	Variables hard to define	We have now received our main datasets and familiarized ourselves with them. The risks related to the data have come more unlikely and are better under control than in the beginning of the project.	Probable	Critical
A2	Part of the data unreliable when considering long time periods.		Likely	Marginal
B	**Validation of the model**			
B1	No data available related to contracts with time to maturity up to six years, so the validation of the model will be hard for longer time periods.	This has been one of the biggest challenges related to the modeling from beginning. We are prepared to do mostly qualitative analysis based on the model instead of exact suggestions.	Probable	Critical
C	**Problems with practicalities**			
C1	Time limit of the course	By now there are no vital problems in this area. We are aware that any hazard event (e.g. long periods of illness) or lack of sufficient communication may change this very suddenly.	Unlikely	Critical
C2	Inefficiency due to big group.		Probable	Marginal
D	**Relevance of the results**			
D1	Usefulness of our results to out client.	If the project does not fulfill its purpose, it will not be functional. To prevent this we have regularly been in contact with our client and have tried to find the most specific and important features of our project compared to previous work in the field.	Unlikely	Catastrophic
D2	Extrapolation of the model, improbable events in market structure		Unlikely	Critical

Figure 3: The updated versions of the risks that were presented in the project plan presentation.

model. Even though the final idea is to add the change of the contract as an external variable, in this model it is treated as an endogenous variable.

Considerably better results can be achieved by using a CVAR model. The Figure 2 shows a CVAR model with the same variables as in the VAR model except the time to maturity. The residuals in this model are considerably better behaved — in a sense of autocorrelation and normality — than the ones in the VAR model. This suggests that we are aiming for CVAR models in the final report.

Our next goal with modeling is to select the variables to be included in the model more carefully. Only preliminary validation of the models has been done by now. Most of the residuals with CVAR model are independently normally distributed but overall test are not made yet.

Probability	Severity			
	Catastrophic	Critical	Marginal	Minor
Likely			**A2**	
Probable		**A1, B1**	**C2**	
Unlikely	**D1**	**C1, D2**		
Remote				

Figure 4: Risk matrix related to the Figure 3.

Current Status of the Project

(a) Schedule and Dividing Jobs

The project runs quite well on schedule. By now we have examined the data and made required preparations. All datasets are modified to the week level, and descriptive analysis related to the stationarity and properties of the data are made. We have also made the codes for the estimation of the model. Now our main focus is in selecting the data and discussing the limitations. In addition, we have started to write parts of the final report. On the other hand, the schedule for the rest of the project is hard so there is no leeway in the case of major obstacles.

By now we have not divided tasks largely. Because of the special characteristics of our project it has been necessary that we all understand the background related to both electricity derivative markets and VAR models extensively. In future we will define the responsibilities more clearly for the final report. Arno, as the project manager, will be responsible for the overall picture. He also has mostly implemented our model into Matlab, thanks to his previous experience. Bahare, Kaisa and Michail will divide writing tasks more in detail based on their special know-how and interest.

(b) Risks

The Figure 3 presents an updated version of our risks and the Figure 4 shows the corresponding risk matrix. All risks considered are quite important because we did not want to waste our time by preparing ourselves for negligible or extreme risks. The risks related to the quality of data (group A) are well under control but the risks in group D, the relevance of results, are still as important as in the beginning of the project.

(c) Tasks

Our next big goal is to develop the model further. Some additional variables will be discussed to capture the special characteristics of electricity markets. The model validation is done contemporaneously to figure out the problems of the model. During two following weeks some intensive work with these topics and background parts of the final report will be done. The more in-depth analysis of the results and discussion about the future scenarios of electricity markets in general will be added to our todo-list after that.

References

Povh, M. (2009). *Stochastic Modelling of Long-Term Electricity Forward Prices*. PhD thesis, University of Ljubljana, faculty of electrical engineering.

Povh, M. and Fleten, S. (2009). Modeling Long-Term Electricity Forward Prices. *IEEE Transactions on Power Systems*, 24(4):1649–1656.

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